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STABILITY OF A FILM FLOWING DOWN ALONG AN OSCILLATING SURFACE

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The linear approximation for a harmonically oscillating surface is used to obtain the condition of flow stability for a liquid film.

The flow of films in heat and mass transfer devices is nearly always accompanied by wave phenomena at the gas—liquid boundary. The waves considerably affect the transfer processes, and, whenever possible, various adaptations are used which assist the formation of waves or turbulence of the liquid. For example, the vibration of a straight surface can, according to the experimental data [1], lead to an increase of the heat-transfer coefficient by 400%, in comparison with the usual gravitational flow. It is therefore of interest to determine the transition from the waveless regime of the flow to the laminar-wave regime, and then to the turbulent regime, i.e., it is necessary to establish the limits of stability of the particular flow regime in question.

Let us assume that a film of viscous incompressible liquid flows down along a sloped surface which oscillates in its own plane with velocity $V_0 \cos\omega_* \tau$ (Fig. 1). The problem is described by the system of equations

$$\nu \frac{\partial^2 v_1}{\partial x_2^2} + g \sin\gamma = \frac{\partial v_1}{\partial \tau}; \quad \frac{\partial p_d}{\partial x_2} = \rho g \cos\gamma; \quad \frac{\partial p_d}{\partial x_3} = 0. \tag{1}$$

In addition, we use the conditions of sticking at the wall, and the absence of tangential stress at the free surface:

$$p_d(0) = p_{atm}; \; v_1|_{x_2=d} = V_0 \cos\omega_*\tau; \quad \left.\frac{\partial v_1}{\partial x_2}\right|_{x_2=0} = 0. \tag{2}$$

By solving the system of equations (1) and (2), we determine the unperturbed flow of the layer in the form

$$u_0 = \frac{1}{2} \operatorname{Re}\operatorname{Fr}^{-1}(1 - y^2)\sin\gamma + \frac{1}{2} \exp(i\omega t) \frac{\operatorname{ch}(1+i)\beta y}{\operatorname{ch}(1+i)\beta} + \frac{1}{2}\exp(-i\omega t)\frac{\operatorname{ch}(1-i)\beta y}{\operatorname{ch}(1-i)\beta};$$

$$p = \frac{y}{\operatorname{Fr}} \cos\gamma + p_a \tag{3}$$

or

$$u_0 = \frac{1}{2} \operatorname{Re}\operatorname{Fr}^{-1}(1 - y^2)\sin\gamma + A\cos(\omega t - t_\alpha). \tag{4}$$

Here $\operatorname{Re} = V_0 d/\nu$ is the vibrational Reynolds number, $\operatorname{Fr} = \frac{V_0^2}{gd}$, vibrational Froud number; $2\beta^2 = \omega\operatorname{Re}$;

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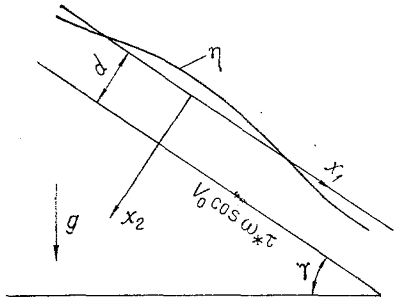

Fig. 1. The flow pattern.

$$A = \left(\frac{\operatorname{sh}^2 \beta y + \cos^2 \beta y}{\operatorname{sh}^2 \beta + \cos^2 \beta} \right)^{1/2},$$

amplitude of instantaneous velocity; and

$$t_\alpha = \arcsin \left(\frac{1}{A} \; \frac{\operatorname{ch} \beta y \cos \beta y \operatorname{sh} \beta \sin \beta}{\operatorname{sh}^2 \beta + \cos^2 \beta} - \frac{\operatorname{sh} \beta y \sin \beta y \operatorname{ch} \beta \cos \beta}{\operatorname{sh}^2 \beta + \cos^2 \beta} \right),$$

phase shift of velocity pulsations in the liquid relative to the oscillations of the wall.

We introduce the length $\delta = (\nu/\omega_*)^{1/2}$, which characterizes the oscillating flow. Therefore, we can have several possibilities: The quantity δ can be small, large, or comparable to the thickness d of the flowing film.

We consider the case $\delta \ll d$ which corresponds to $\beta \gg 1$ and is realized, e.g., for high frequencies of oscillations of the wall. It follows from Eq. (4) that the velocity pulsations decay with increasing distance from the wall as $A \sim \exp(-\beta(1 - y))$, and they will, therefore, be localized in a narrow near-wall region. Across most of the thickness, u_0 is parabolic. In the problem of stability of a flow with a dividing boundary, the instability mechanism is associated with the formation of surface waves generated by the perturbation of the free surface. It is therefore natural to assume that the near-wall velocity pulsations cannot interact with the perturbations of the free surface, and the stability conditions will be the same as in the absence of vibrations. We introduce

$$\mathrm{Re}_c = \frac{\mathrm{Re}^2 \sin \gamma}{2\mathrm{Fr}} \equiv \frac{g \rho^2 d^3 \sin \gamma}{3 \mu^2},$$

and obtain [2]

$$\mathrm{Re}_c < \frac{5}{6} \operatorname{ctg} \gamma. \tag{5}$$

We turn to the opposite case when $\delta \gg d$ $(\beta \ll 1)$, which is realized for low frequencies of oscillation of the wall. The smallness of frequency indicates that the velocity varies slowly with time, and in Eq. (1) one can therefore neglect the derivative $\partial v_1/\partial \tau$. Consequently, the flow can be assumed stationary. It follows from Eq. (4) that in this regime the film flows down along the solid surface and oscillates with it as a whole, i.e., it moves with velocity

$$u_0 = \cos \omega t + \frac{1}{2} \mathrm{Re}\, \mathrm{Fr}^{-1} (1 - y^2) \sin \gamma \tag{6}$$

at each moment of time. As for a stationary wall, the stability condition can be written in the form (5).

Finally, we consider the case $\delta \sim d$, i.e. $\beta \sim 1$ or $\omega_* \sim \nu/d^2$. Let us assume that at some moment of time the principal flow (3) suffers a small two-dimensional perturbation $\hat{u} = u_0 + u'$; $\hat{v} = v'$; $\hat{p} = p_0 + p'$. We introduce the flow function by

$$u' = \frac{\partial \psi}{\partial y}; \; v' = -\frac{\partial \psi}{\partial x}$$

and assume that

$$\psi = \varphi(y, t)\exp(i\alpha x); \quad \eta = \eta(t)\exp(i\alpha x); \tag{7}$$
$$p' = \tilde{p}(y, t)\exp(i\alpha x).$$

Here η is the geometry of the perturbed surface. By linearizing, in the usual fashion, the equations of motion and the continuity equation, and eliminating the pressure $\tilde{p}$, we obtain, by using dynamic and kinematic boundary conditions, the description of the perturbed flow of the liquid layer:

$$\frac{\partial^4\varphi}{\partial y^4} - 2\alpha^2\frac{\partial^2\varphi}{\partial y^2} + \alpha^4\varphi = \mathrm{Re}\left[\frac{\partial}{\partial t}\left(\frac{\partial^2\varphi}{\partial y^2} - \alpha^2\varphi\right) + \right.$$
$$\left. + i\alpha u_0\left(\frac{\partial^2\varphi}{\partial y^2} - \alpha^2\varphi\right) - i\alpha\varphi\frac{\partial^2 u_0}{\partial y^2}\right]; \tag{8}$$

$$-i\alpha\varphi(0, t) = i\alpha\eta u_0(0, t) + \frac{d\eta}{dt}; \tag{9}$$

$$\varphi = 0, \quad \frac{\partial\varphi}{\partial y} = 0 \text{ for } y = 1;$$
$$\eta\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2\varphi}{\partial y^2} + \alpha^2\varphi = 0 \text{ for } y = 0;$$
$$-i\alpha^3\mathrm{Re}\,\eta = i\alpha\,\mathrm{Re}\,\mathrm{Fr}^{-1}\eta\cos\gamma - 2\alpha^2\frac{\partial\varphi}{\partial y} + \frac{\partial^3\varphi}{\partial y^3} - \tag{10}$$
$$-\alpha^2\frac{\partial\varphi}{\partial y} - \mathrm{Re}\frac{\partial^2\varphi}{\partial t\partial y} - i\alpha\,\mathrm{Re}\,u_0\frac{\partial\varphi}{\partial y} \text{ for } y = 0.$$

Equation (8) is the analogue of the Orr—Sommerfeld equation for the nonstationary case. According to [3], we seek the solution of the system (8)-(10) in the form

$$\varphi(y, t) = f(y, t)\exp(\tilde{\mu}t); \quad \eta(t) = N(t)\exp(\tilde{\mu}t), \tag{11}$$

where f(y, t) and N(t) are periodic functions of time, and $\tilde{\mu}$ is a complex number.

First of all, we consider long-wavelength perturbations $\alpha \ll 1$ since the short-wavelength perturbations are quenched by the surface tension. We represent all quantities in the form of series in terms of

$$f = f_0 + \alpha f_1 + \alpha^2 f_2 + \ldots; \quad N = N_0 + \alpha N_1 + \alpha^2 N_2 + \ldots;$$
$$\tilde{\mu} = \beta_0 + \alpha\beta_1 + \alpha^2\beta_2 + \ldots$$

We now consider the consecutive approximations in terms of α. In the first approximation, we obtain

$$\frac{\partial^4 f_0}{\partial y^4} = \mathrm{Re}\frac{\partial^3 f_0}{\partial t\partial y^2} + \beta_0\mathrm{Re}\frac{\partial^2 f_0}{\partial y^2}; \quad \frac{dN_0}{dt} + \beta_0 N_0 = 0;$$
$$f_0 = 0; \quad \frac{\partial f_0}{\partial y} = 0 \text{ for } y = 1;$$
$$\frac{1}{\mathrm{Re}}\frac{\partial^3 f_0}{\partial y^3} - \frac{\partial^2 f_0}{\partial t\partial y} - \beta_0\frac{\partial f_0}{\partial y} = 0 \text{ for } y = 0;$$
$$\frac{\partial^2 f_0}{\partial y^2} - N_0\mathrm{Re}\,\mathrm{Fr}^{-1}\cos\gamma + i\beta^2 N_0\left(\frac{\exp(i\omega t)}{\mathrm{ch}(1+i)\beta} - \frac{\exp(-i\omega t)}{\mathrm{ch}(1-i)\beta}\right) = 0$$
$$\text{for } y = 0.$$

Using the periodicity conditions for the functions N_0 and f_0 gives

$$\beta_0 = 0; \quad N_0 = \mathrm{const} = 1;$$
$$f_0 = \frac{1}{2}\mathrm{Re}\,\mathrm{Fr}^{-1}\sin\gamma\,(1-y)^2 + \mathrm{Real}\left\{\exp(i\omega t)\times\right.$$
$$\left.\times\left[\frac{1}{\mathrm{ch}^2(1+i)\beta} - \frac{\mathrm{ch}(1+i)\beta y}{\mathrm{ch}(1+i)\beta} + \frac{\mathrm{th}(1+i)\beta}{\mathrm{ch}(1+i)\beta}\,\mathrm{sh}(1+i)\beta y\right]\right\}.$$

In the second approximation,

$$\beta_1 + \frac{dN_1}{dt} + iu_0(0,\ t) = -if_0(0,\ t).$$

Since β_1 is a number and $N_1(t)$ is a periodic function, we have

$$\beta_1 = -i\,\mathrm{Re}\,\mathrm{Fr}^{-1}\sin\gamma;\ \ N_1(t) = -\frac{1}{2\omega}\left[\frac{\exp(i\omega t)}{\mathrm{ch}^2(1+i)\beta} - \frac{\exp(-i\omega t)}{\mathrm{ch}^2(1-i)\beta}\right]$$

and for f_1 we have the equation and boundary condition

$$\frac{\partial^4 f_1}{\partial y^4} - \mathrm{Re}\frac{\partial^3 f_1}{\partial t\partial y^2} = i\,\mathrm{Re}\left(u_0\frac{\partial^2 f_0}{\partial y^2} - f_0\frac{\partial^2 u_0}{\partial y^2}\right) + \beta_1\,\mathrm{Re}\frac{\partial^2 f_0}{\partial y^2};\tag{12}$$

$$f_1 = 0;\ \ \frac{\partial f_1}{\partial y} = 0\ \text{ for }\ y = 1;$$

$$\frac{\partial^2 f_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial y^2}N_1 = 0\ \text{ for }\ y = 0;$$

$$i\,\mathrm{Re}\,\mathrm{Fr}^{-1}\cos\gamma + \frac{\partial^3 f_1}{\partial y^3} - i\,\mathrm{Re}\,u_0\frac{\partial f_1}{\partial y} -$$

$$-\mathrm{Re}\frac{\partial^2 f_1}{\partial t\partial y} - \beta_1\,\mathrm{Re}\frac{\partial f_0}{\partial y} = 0\ \text{ for }\ y = 0.\tag{13}$$

The solution of equation (12) will be sought in the form

$$f_1 = \Psi_1(y)\exp(2i\omega t) + \Psi_2(y)\exp(-2i\omega t) + \Psi_3(y).$$

We now turn to the following expansion in terms of α (the terms which are coefficients of α^2): To determine β_2 from the equation

$$\frac{dN_2}{dt} = -\beta_2 - \beta_1 N_1 - i(f_1(0,\ t) + N_1 u_0(0,\ t))\tag{14}$$

under the condition that β_2 is a number and N_2 is a periodic function, we only need to find the dependence $\Psi_3(y)$:

$$\Psi_3(y) = \gamma_1 y^3 + \gamma_2 y^2 + \gamma_3 y + \gamma_4 + i\,\mathrm{Re}\left\{\frac{k^2}{6}\left(y^4 - \frac{y^5}{5}\right) +\right.$$

$$+\frac{i}{4\beta^2(\mathrm{ch}\,2\beta + \cos 2\beta)}\left[\frac{\mathrm{ch}(1+i)\beta y}{\mathrm{ch}(1-i)\beta} - \frac{\mathrm{ch}(1-i)\beta y}{\mathrm{ch}(1+i)\beta}\right] +$$

$$+\frac{i}{16\beta^2(\mathrm{ch}\,2\beta + \cos 2\beta)}[(\mathrm{sh}\,2\beta y + i\sin 2\beta y)\,\mathrm{th}(1+i)\beta -$$

$$\left.-(\mathrm{sh}\,2\beta y - i\sin 2\beta y)\,\mathrm{th}(1-i)\beta]\right\};\ \ k = \frac{\mathrm{Re}}{2\,\mathrm{Fr}}\sin\gamma;$$

$$\gamma_1 = -\frac{i\,\mathrm{Re}}{6\mathrm{Fr}}\cos\gamma - \frac{ik^2}{3}\mathrm{Re};\ \ \gamma_2 = ik^2\mathrm{Re};$$

$$\gamma_3 = \frac{3i\,\mathrm{Re}(\mathrm{ch}\,2\beta\sin 2\beta + \mathrm{sh}\,2\beta\cos 2\beta)}{\beta(\mathrm{ch}\,2\beta + \cos 2\beta)^2} - \frac{3}{2}ik^2\,\mathrm{Re} + \frac{i\,\mathrm{Re}}{2\,\mathrm{Fr}}\cos\gamma;$$

$$\gamma_4 = \frac{3i\,\mathrm{Re}}{4\beta(\mathrm{ch}\,2\beta + \cos 2\beta)^2}\left[\frac{1}{\beta}\mathrm{sh}\,2\beta\sin 2\beta - \mathrm{ch}\,2\beta\sin 2\beta -\right.$$

$$\left.-\mathrm{sh}\,2\beta\cos 2\beta\right] - \frac{1}{3}i\,\mathrm{Re}\,\mathrm{Fr}^{-1}\cos\gamma + 0.7ik^2\,\mathrm{Re}.$$

We now obtain from Eq. (14)

$$\beta_2 = \mathrm{Re}\,\mathscr{F}(\beta) + \frac{2\,\mathrm{Re}^3\sin^2\gamma}{15\,\mathrm{Fr}^2} - \frac{\mathrm{Re}\cos\gamma}{3\,\mathrm{Fr}},\tag{15}$$

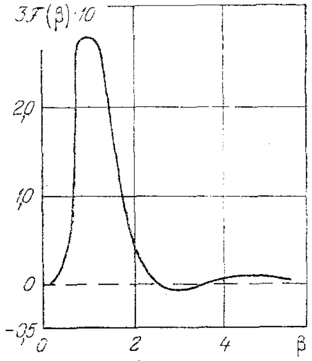

Fig. 2. The variation of the function $\mathcal{F}(\beta)$ with increasing β.

where

$$\mathcal{F}(\beta) = \frac{3}{4\beta(\operatorname{ch} 2\beta + \cos 2\beta)^2}\left[\frac{1}{\beta}\operatorname{sh} 2\beta \sin 2\beta - \right.$$
$$\left. - \operatorname{ch} 2\beta \sin 2\beta - \operatorname{sh} 2\beta \cos 2\beta\right].$$

The stability of flow is determined by the sign of β_2: for $B_2 < 0$ the flow is stable, and for $\beta_2 > 0$ it is unstable or, in other words, the flow is stable if

$$3\,\mathrm{Fr}\,\mathcal{F}(\beta) + \frac{6}{5}\mathrm{Re}_c \sin\gamma - \cos\gamma < 0. \tag{16}$$

In the opposite case, the flow is unstable and the perturbations increase exponentially.

It is seen from Fig. 2 that $\mathcal{F}(\beta)$ differs considerably from zero only in a very narrow region of frequencies $\beta \sim 1$; i.e., $\omega^* \sim \nu/d^2$, where the vibrations of the wall are most favorable for the loss of stability. When the vibrations are absent ($V_0 \equiv 0$), Eq. (16) makes it possible to obtain the known result for the stationary layer [2]: The flow is unstable for $\mathrm{Re}_c < \frac{5}{6}\operatorname{ctg}\gamma$.

In the situation when the angle of inclination $\gamma = 0$, i.e., the flow under the action of gravity is absent, we obtain the conditions for the stability of a horizontal layer [3]: $\mathrm{Fr}^{-1} > 3F(\beta)$. For a vertical surface, $\gamma = \pi/2$, and the condition (16) takes the form

$$\mathrm{Re}_c < -\frac{5}{2}\mathrm{Fr}\,\mathcal{F}(\beta). \tag{17}$$

Hence, it is seen that on a stationary wall the motion is always stable. Since the function $F(\beta)$ changes sign, for a vibrating surface the stable regime can exist for some values of amplitudes and frequencies of oscillations.

We consider the case of small-amplitude oscillations of the velocity of the wall ($\mathrm{Fr} \ll 1$). It then follows from Eq. (16), in view of the fact that function $\mathcal{F}(\beta)$ is bounded, that, independently of the frequency of oscillations, the condition $\mathrm{Re}_c < \frac{5}{6}\operatorname{ctg}\gamma$ is satisfied. In other words, the vibrations of the wall do not affect the stability of the film.

The results obtained in the present work are confirmed indirectly by the experimental investigations [4], which studied the pattern of perturbations on the surface of a flowing film for an oscillatory spraying density. In our case, by going over to a coordinate system fixed to the wall, and introducing the volume flow rate of the liquid $Q = \int_0^d u_0 dy$, we obtain a solution which describes the flow of the film along a stationary surface, but with a pulsating flow rate.

NOTATION

x_1, x_2, and x_3 are the orthogonal Cartesian coordinates; v_1, v_2, v_3, dimensional components of the velocity vector; d, thickness of the film; V_0, amplitude of the velocity of wall oscillations; $x = x_1/d$, $y = x_2/d$, $z = x_3/d$, dimensionless Cartesian coordinates; $u_0 = v_1/V_0$, dimensionless velocity of unperturbed flow; p_d, dimensional pressure in the liquid; p_{atm}, gas pressure at the free surface; $p_\alpha = p_{atm}/\rho V_0^2$, dimensionless pressure of gas at the free surface; ρ, density; ν, kinematic viscosity; τ, time; $t = \tau V_0/d$, dimensionless time; ω_*, frequency of oscillations of the wall; $\omega = \omega_* d/V_0$, dimensionless frequency; and $\hat{u}$, $\hat{v}$, $\hat{p}$, components of the velocity vector and pressure in the perturbed flow.

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NUMERICAL MODELING OF AN EXPLOSION PLASMA GENERATOR, TAKING INTO ACCOUNT RADIATIVE ENERGY TRANSPORT AND EVAPORATION OF THE WALLS

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Using a generalized theoretical model, we study the effect of geometrical dimensions and a variety of physical processes on the operation of the explosion plasma generator.

The present work is a continuation of the theoretical study [1, 2] of the explosion plasma generator developed by A. E. Voitenko (Fig. 1). The principles of operation of this device were discussed in sufficient detail in [1-4]. This makes it possible to turn to the formulation of the problem of numerical modeling of the explosion plasma generator.

In the course of operation of the generator, a detonation wave, by passing over the explosives, accelerates a metal plate up to the velocity 5-6 km/sec [2-4]. The phase velocity of the point on the line of contact of the plate and segment will increase (since $U_\Phi = V/\sin\varphi$) as the plate approaches the exit aperature and, from some moment of time, it will exceed the speed of sound c_0 in the metal plate and in the spherical segment. Therefore, the perturbations from the contact line (metal deformations, the melt from the walls, etc.) will not propagate upwards along the flow, and affect the motion of the plate and of the gas. It is probable that this can explain the experimental facts that the plate remains flat during the motion inside the segment (this is seen on the x-ray pulsed picture shown in [5]). Under the action of the detonation wave, shock waves propagate and are reflected from the surface of the plate, and cause its expansion and compression. However, the amplitude of these oscillations is small (it is smaller than the proper thickness of the plate), and is much smaller than the path length of the plate inside the segment. It is known [6] that a compressible plate gains velocity discontinuously, and on incompressible plate gains velocity in a smooth fashion. However, the difference of instantaneous velocities of motion is in

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